4 For each of the following, state the implied domain and range:

a $f(x) = \sqrt{x-4}$ **b** $f(x) = \sqrt{4-x}$

c $f(x) = 2\sqrt{x-2}+3$ **d** $f(x) = \frac{1}{x-4}$

e $f(x) = \frac{1}{x-4}+3$ **f** $f(x) = \frac{3}{x+2}-3$

Implied domain: the function tells us that certain values can not be used. For example, the square root. They usually limit/rule out the negative values within them.

For the hyperbolas, there are undefined values when the denominator is 0.

a) $y = \sqrt{x-4}$

if $x = 2$, $f(x) = \sqrt{2-4} = \sqrt{-2}$ ✗

so 2 and no domain!

$x = 5$, $\sqrt{5-4} = \sqrt{1}$ ✗

$x = 4$, $\sqrt{4-4} = \sqrt{0} = 0$

$x = 5$, $\sqrt{5-4} = \sqrt{1} = 1$

domain :: $[4, \infty)$ range :: $[0, \infty)$

b) $f(x) = \sqrt{4-x}$

domain: $(-\infty, 4]$

range :: $[0, \infty)$

c) $\sqrt{x-2}$, $\sqrt{x}$ ✓

$2\sqrt{x-2}$, 3

domain $[2, \infty)$ ✓

range $[3, \infty)$

d) $\frac{1}{x-4}$: $\frac{1}{0}$ ✗

$y = \frac{1}{x}$

$x = 0$

$y = 0$

$R \setminus \{0\}$ or $(-\infty, \infty)$

$R \setminus \{4\}$ or $(-\infty, 4) \cup (4, \infty)$

$x = 4$ and $y = 0$ are the asymptotes

$(-\infty, 4) \cup (4, \infty)$

$y = \frac{1}{x} + 0$

$x = 0$ and $y = 0$

domain/range: $x \neq 4$, $y \neq 0$

e) $\frac{1}{x-4} + 3$

$x = 4$ impacts the implied DOMAIN (x-direction)

$y = 3$ impacts the implied RANGE (y-direction)

domain :: $R \setminus \{4\}$

range :: $R \setminus \{3\}$

$(-\infty, 4) \cup (4, \infty)$

$(-\infty, 3) \cup (3, \infty)$

f) $\frac{3}{x+2} - 3$

domain $R \setminus \{-2\}$

range $R \setminus \{-3\}$

$+\frac{1}{x}$

$-\frac{1}{x}$

SE

$f(x) = \begin{cases} 2x-1, & \text{for } x \geq 0 \\ 3x+1, & \text{for } -2 \leq x < 0 \\ 3x+1, & \text{for } x < -2 \end{cases}$

tution

$y = mx + c$

$y = x$ $x < 0$

$y = 3$ $x \geq 0$

1 Sketch the graph of each of the following functions and state its range:

a $h(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$ **b** $h(x) = \begin{cases} x-1, & x \geq 1 \\ 1-x, & x < 1 \end{cases}$ **c** $h(x) = \begin{cases} -x, & x \geq 0 \\ x, & x < 0 \end{cases}$

d $h(x) = \begin{cases} x+1, & x \geq 0 \\ 1-x, & x < 0 \end{cases}$ **e** $h(x) = \begin{cases} x, & x \geq 1 \\ \frac{1}{x}, & x < 1 \end{cases}$

$y = 1 - x$

$0 = 1 - x$

$x = 1$

a)

range $[0, \infty)$

b)

c)

$y = 2 - x$

x-int

$y = 0$

$0 = 2 - x$

$x = 2$

y-int $x = 0$

$y = 2 - 0$

$y = 2$

y-int + gradient

$y = 2 - x$

y-int $(0) = 2$

gradient $= -1$

y-int $= 2$

gradient $= m = \frac{y_2 - y_1}{x_2 - x_1}$, $\frac{rise}{run} = \frac{-1}{1}$

then 1 point & the grad

$\frac{rise}{run} = \frac{2}{3}$

$0 = \frac{2}{3}x + 2$

$3 \times -2 = \frac{2}{3}x \times 3$

$\frac{-6}{2} = \frac{2x}{2}$

$x = -3$

4 Sketch the graph of the following function and state its range:

$h(x) = \begin{cases} x^2+1, & x \geq 0 \\ 1-x, & x < 0 \end{cases}$

$y = x^2 + 1$

$y = 1 - x$

parabola

linear

2nd hour

domain & range

a $f(x) = \begin{cases} x+3, & x \leq -3 \\ x^2-9, & -3 \leq x \leq 3 \\ x-3, & x > 3 \end{cases}$

$y = x + 3$

$0 = x + 3$

$x = -3$

$y = x^2 - 9$

$0 = x^2 - 9$

$\sqrt{x^2} = \sqrt{9}$

$x = \pm 3$ ✓

domain $[-9, \infty)$

range $(-\infty, \infty)$ or $\mathbb{R}$

b $g(x) = \begin{cases} x+3, & x \leq -1 \\ x-2, & 1 \leq x \leq 5 \\ 2x-1, & x \geq 5 \end{cases}$

only

d $f(x) = \begin{cases} \frac{1}{x+3}, & x < -3 \\ (x+2)^2, & x \geq -1 \end{cases}$

$y = \frac{1}{x+3}$

$x = -3$

$y = (x+2)^2$

$x = -2$

T. P

range $(-\infty, 0) \cup [1, \infty)$

domain $(-\infty, 0) \cup [-1, \infty)$

8 Specify the function represented by this graph:

$f(x) = \begin{cases} y = x+3 & -3 \leq x < 1 \\ y = -x+1 & -1 \leq x \leq 1 \\ y = -1 & 1 \leq x \leq 4 \end{cases}$

2nd line

$(-1, 2)$, $(2, -1)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-2}{2-(-1)}$

$= \frac{-3}{3} = -1$

$y = mx + c$

$2 = -1 \times -1 + c$

$2 = 1 + c$

$c = 1$

$y = mx + c$

$(-3, 0)$, $(1, 2)$

x_1, y_1

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{2-0}{1-(-3)}$

$= \frac{1}{2}$

$y = \frac{1}{2}x + c$

sub in eq. of 2 pts.

$2 = \frac{1}{2} \times 1 + c$

$2 = \frac{1}{2} + c$

$c = \frac{3}{2}$ ✓

$y = \frac{1}{2}x + \frac{3}{2}$

$y = mx + c$

$(-2, -1)$, $(1, 2)$

x_1, y_1

x_1, y_1

$1 = m \times -2 + c \ldots$ ①

$2 = m \times 1 + c \ldots$ ②

$2 = m + c$

$1 = -2m + c$

$c = 2m + 1$

sub c into eq. ①

$1 = -2m + (2m + 1)$

$1 = -2m + 2m + 1$

$1 = m + 1$

$0 = m$

$m = 0$

sub m into eq. ②

$c = 2m + 1$

$c = 2 \times 0 + 1$

$c = 1$

$y = 0x + 1$

$y = 1$

3rd line

$(2, -1)$, $(4, -3)$

(x_1, y_1) (x_2, y_2)

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-1}{4-2} = \frac{-3}{2}$

$y = mx + c$

$-1 = -\frac{3}{2} \times 2 + c$

$-1 = -3 + c$

$2 = -1 + c$

$c = 2$

$y = -\frac{3}{2}x + 2$

$-2m = -m$

Substitution

$1 = -2m + c$

$c = 2m + 1 \ldots$ ①

sub c into eq. ②

$2 = m \times -1 + (2m + 1)$

$2 = -m + 2m + 1$

$2 = m + 1$

$m = 1$

sub m into eq. ①

$c = 2m + 1$

$c = 2 \times 1 + 1$

$c = 3$

$y = 1x + 3$

$y = x + 3$

Elimination

$0 = 1$

$1 = -2m + c$

$-(2 = m + c)$

$-1 = -m + 0$

$m = 1$

$2 = 1 + c$

$c = 2 - 1 = 1$

$y = x + 1$