

Example 66

Find all the values of θ , correct to the nearest minute if necessary, given that $0^\circ \leq \theta \leq 360^\circ$.

a) $\tan 2\theta = 1$

Let $u = 2\theta$
 $0^\circ \leq \theta \leq 360^\circ$
 $0^\circ \leq 2\theta \leq 720^\circ$
 $0^\circ \leq u \leq 720^\circ$
 $\tan(u) = 1$
 $u = 45^\circ, 225^\circ, 405^\circ, 585^\circ$
 $2\theta = 45^\circ, 225^\circ, 405^\circ, 585^\circ$
 $\theta = \frac{45^\circ}{2}, \frac{225^\circ}{2}, \frac{405^\circ}{2}, \frac{585^\circ}{2}$
 $= 22.5^\circ, 112.5^\circ, 202.5^\circ, 292.5^\circ$

b) $2 \cos 2\theta - 1 = 0$

$2 \cos 2\theta = 1$
 $\cos(2\theta) = \frac{1}{2}$ Let $u = 2\theta$
 $\cos(u) = \frac{1}{2}$
 $u = 60^\circ, 300^\circ$
 as $0^\circ \leq 2\theta \leq 720^\circ$
 $0^\circ \leq u \leq 720^\circ$
 also includes
 $u = 60^\circ + 360^\circ, 300^\circ + 360^\circ$
 $= 420^\circ, 660^\circ$
 $u = 60^\circ, 300^\circ, 420^\circ, 660^\circ$
 $2\theta = 60^\circ, 300^\circ, 420^\circ, 660^\circ$
 $\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$ ✓

c) $5 \sin\left(\frac{\theta}{2}\right) = 4$

$\sin\left(\frac{\theta}{2}\right) = \frac{4}{5}$
 as $0^\circ \leq \theta \leq 360^\circ$
 $0^\circ \leq \frac{\theta}{2} \leq 180^\circ$
 Let $u = \frac{\theta}{2}$
 $\sin(u) = \frac{4}{5}$
 $u = 53^\circ 8'$
 $u = 180^\circ - 53^\circ 8'$
 $= 126^\circ 52'$
 $\frac{\theta}{2} = 53^\circ 8', 126^\circ 52'$
 $\theta = 106^\circ 16', 253^\circ 42'$

d) $\sin(\theta - 250^\circ) = \frac{\sqrt{3}}{2}$

$0^\circ \leq \theta \leq 360^\circ$
 $\theta - 250^\circ \rightarrow \theta - 250^\circ$
 $0 - 250^\circ \leq \theta - 250^\circ \leq 360 - 250^\circ$
 $-250^\circ \leq \theta - 250^\circ \leq 110^\circ$
 Let $u = \theta - 250^\circ$
 $-250^\circ \leq u \leq 110^\circ$
 $\sin(u) = \frac{\sqrt{3}}{2}$
 $u = 60^\circ, 180^\circ - 60^\circ$
 + in 1st & 2nd
 $-180^\circ - 60^\circ, -360^\circ + 60^\circ$
 $= -300^\circ < -250^\circ$
 $\therefore X$
 $u = 60^\circ, -240^\circ$
 $\theta - 250^\circ = 60^\circ, -240^\circ$
 $\theta = 310^\circ, 10^\circ$ ✓
 $\theta = 10^\circ, 310^\circ$ ✓

Example 67

2ADV T2 2016 HSC 11g

Solve $\sin\left(\frac{x}{2}\right) = \frac{1}{2}$ for $0 \leq x \leq 2\pi$. (2 marks)

Let the
 graph be

$0 \leq x \leq 2\pi$
 $0 \leq \frac{x}{2} \leq \pi$
 $\sin\left(\frac{x}{2}\right) = \frac{1}{2}$
 Let $u = \frac{x}{2}$
 $\sin(u) = \frac{1}{2}$, $0 \leq u \leq \pi$
 $u = \sin^{-1}\left(\frac{1}{2}\right)$
 $u = 30^\circ, 180^\circ - 30^\circ$
 $u = 30^\circ, 150^\circ$
 $\frac{x}{2} = 30^\circ, 150^\circ$
 $x = 60^\circ, 300^\circ$
 $x = \frac{\pi}{3}, \frac{5\pi}{3}$ ✓

Solve $\sin\left(\frac{x}{2}\right) = \frac{1}{2}$ for $0 \leq x \leq 2\pi$. (2 marks)

$0 \leq x \leq 2\pi$
 $\frac{0}{2} \leq \frac{x}{2} \leq \frac{2\pi}{2}$
 $0 \leq \frac{x}{2} \leq \pi$
 $0 \leq \frac{x}{2} \leq 180^\circ$
 since $\sin\left(\frac{x}{2}\right) = \frac{1}{2}$
 Let $u = \frac{x}{2}$
 $\sin(u) = \frac{1}{2}$
 $u = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$
 As $\sin$ is + in 1st & 2nd quad
 then
 $u = 180^\circ - 30^\circ$ as well
 $u = 30^\circ, 150^\circ$
 $\frac{x}{2} = 30^\circ, 150^\circ$
 $x = 60^\circ, 300^\circ$
 $x = 60^\circ \times \frac{\pi}{180}, 300^\circ \times \frac{\pi}{180}$
 $= \frac{\pi}{3}, \frac{5\pi}{3}$ ✓