

Example 3 1 Evaluate each of the following:

- a** $\sin(3\pi)$ **b** $\cos(-\frac{5\pi}{2})$ **c** $\sin(\frac{7\pi}{2})$ **d** $\cos(3\pi)$
e $\sin(-4\pi)$ **f** $\tan(-\pi)$ **g** $\tan(2\pi)$ **h** $\tan(-2\pi)$
i $\cos(23\pi)$ **j** $\cos(\frac{49\pi}{2})$ **k** $\cos(35\pi)$ **l** $\cos(-\frac{45\pi}{2})$
m $\tan(24\pi)$ **n** $\cos(20\pi)$

From the periodicity of the circular functions:

- $\sin(2k\pi + \theta) = \sin \theta$, for all integers k
- $\cos(2k\pi + \theta) = \cos \theta$, for all integers k
- $\tan(k\pi + \theta) = \tan \theta$, for all integers k .

... -3, -2, -1, 0, 1, 2, 3 ...
 - use formulas
 - to set it back
 to range 0 to 2π .

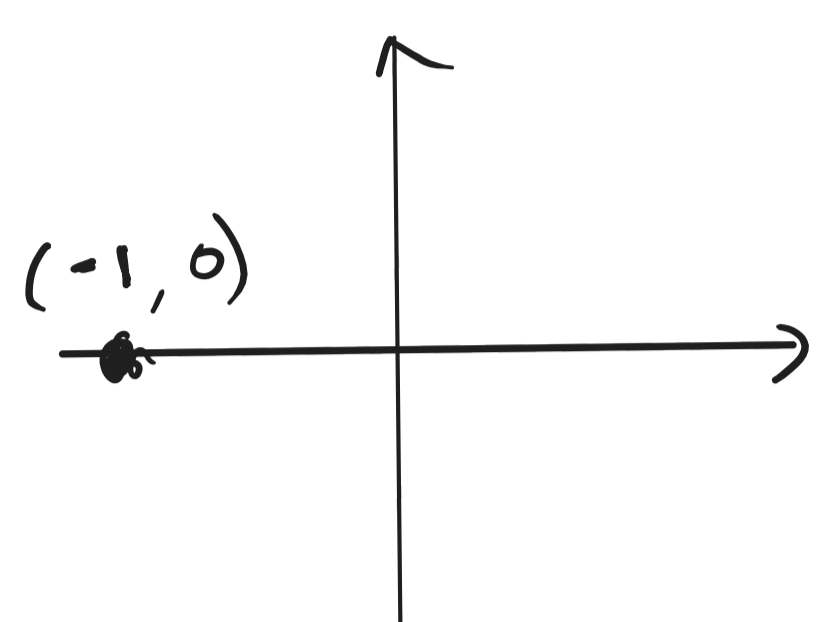
a) $\sin(3\pi)$

$\sin(\theta \pm 2k\pi) = \sin(\theta)$, k is positive natural numbers (1, 2, 3, ...)

$\sin(3\pi + \text{or } - 2 \times 2 \times \pi) = \sin(3\pi - 4\pi) = \sin(-\pi)$ ✗

$\sin(3\pi - 2 \times 1 \times \pi) = \sin(3\pi - 2\pi) = \sin(\pi)$ ✓

$\sin(3\pi) = \sin(\pi) = \frac{0}{1} = 0$

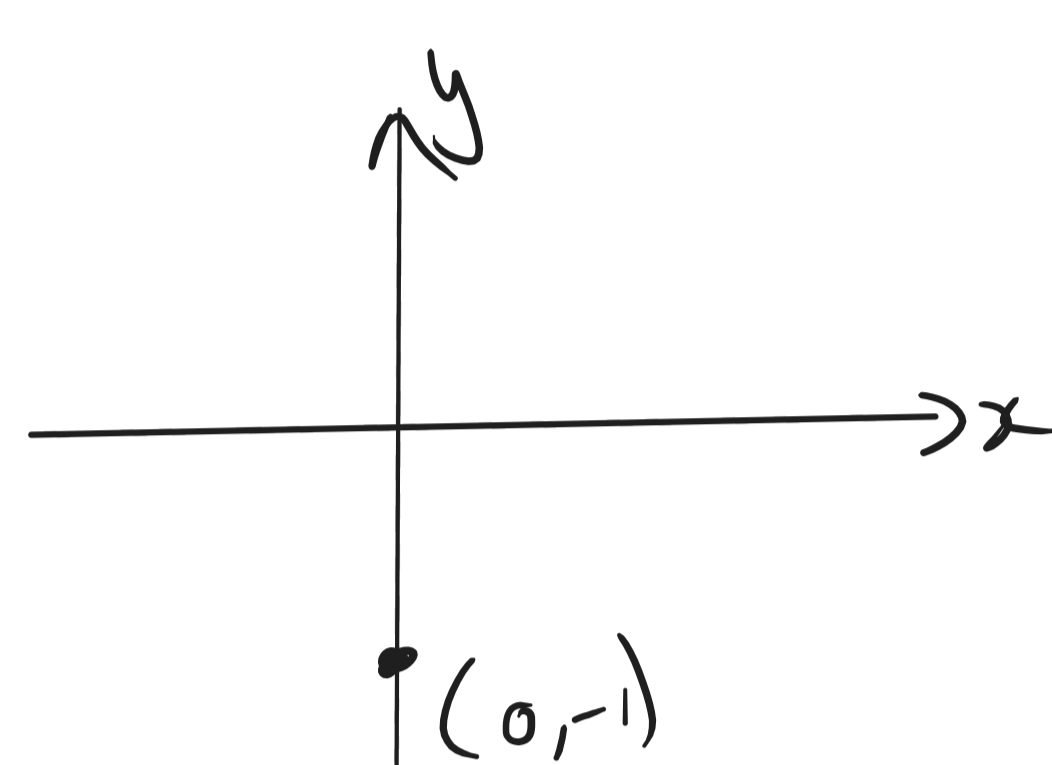


in unit circle
 $\cos(\theta) = x \leftarrow \frac{x}{r}$
 $\sin(\theta) = y \leftarrow \frac{y}{r}$
 $\tan(\theta) = \frac{\sin \theta}{\cos \theta} = \frac{y}{x}$

b) $\cos(-\frac{5\pi}{2}) =$

$\cos(\theta \pm 2k\pi) = \cos(\theta)$

$\cos(-\frac{5\pi}{2} + 2 \times 2 \times \pi) = \cos(-\frac{5\pi}{2} + 4\pi) = \cos(-\frac{5\pi}{2} + \frac{8\pi}{2}) = \cos(\frac{3\pi}{2})$ ✓

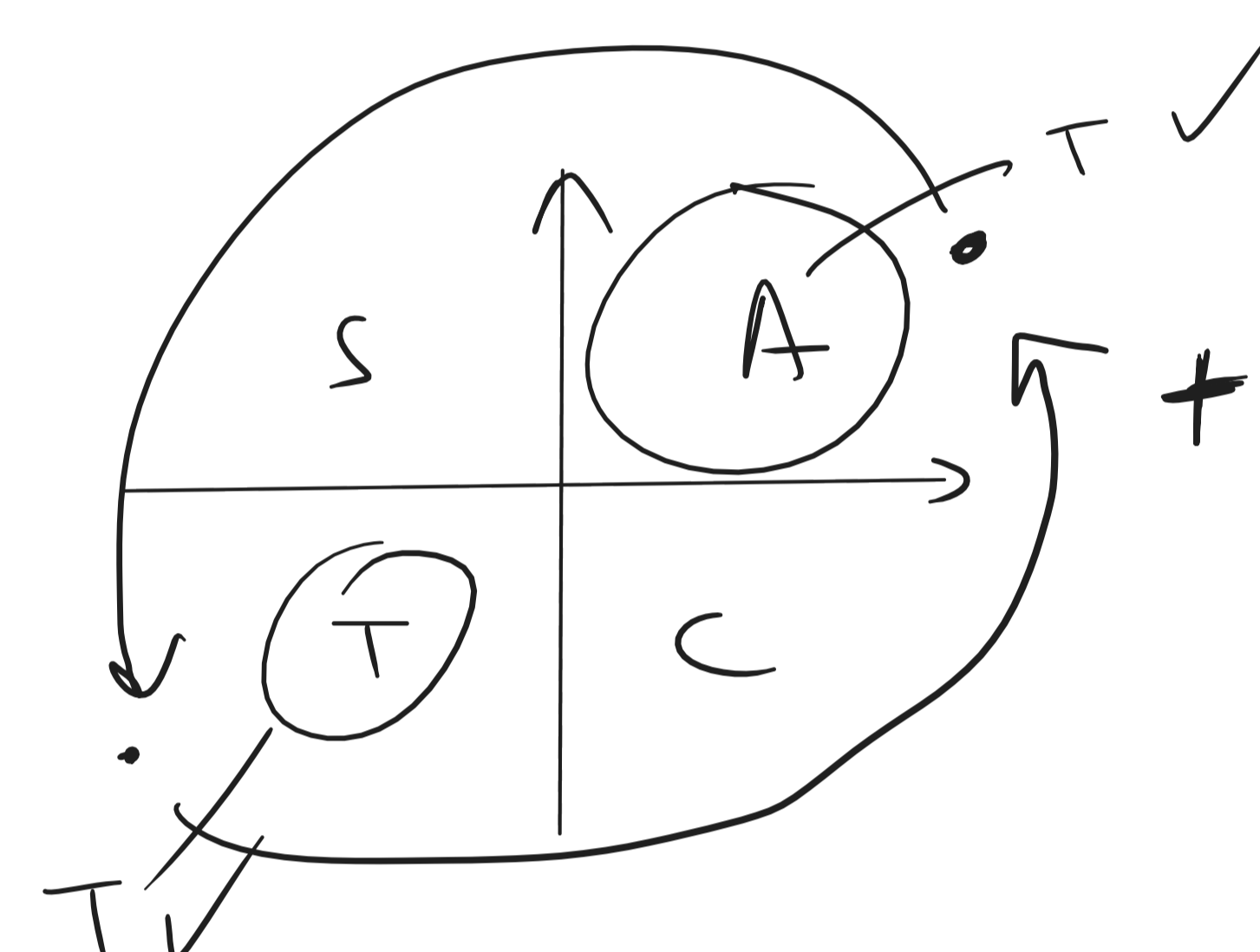
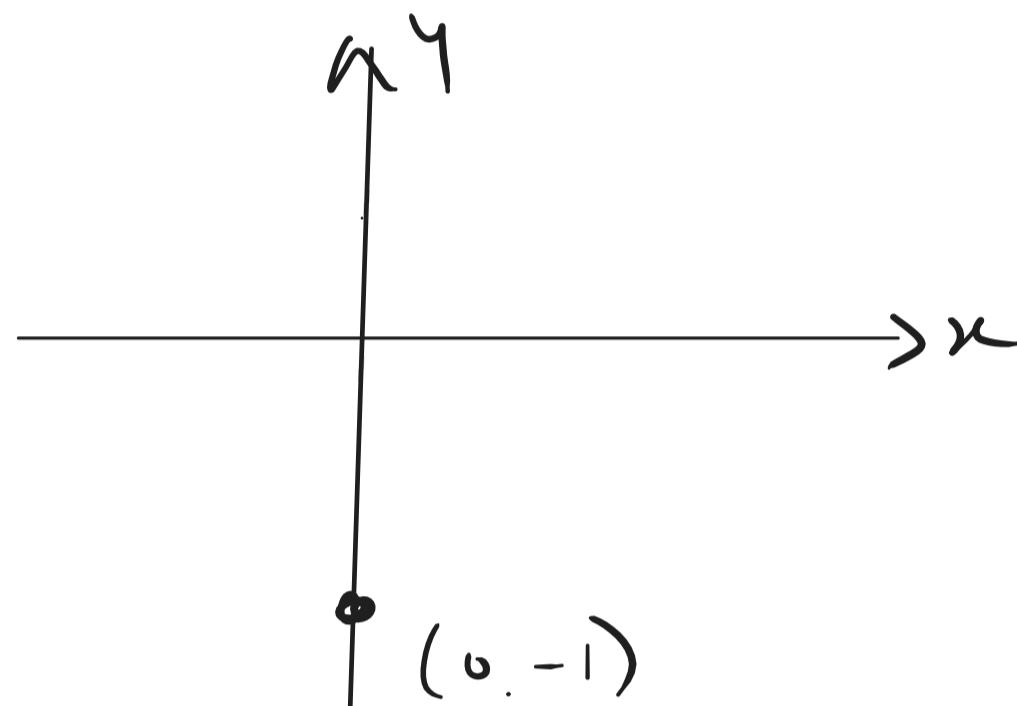


$\cos(\frac{3\pi}{2}) = 0$ ✓

c) $\sin(\frac{7\pi}{2})$

$\sin(\theta \pm 2k\pi) = \sin(\theta)$
 $= \sin(\frac{7\pi}{2} - 2 \times 1 \times \pi) = \sin(\frac{7\pi}{2} - \frac{4\pi}{2}) = \sin(\frac{3\pi}{2})$

$= -1$

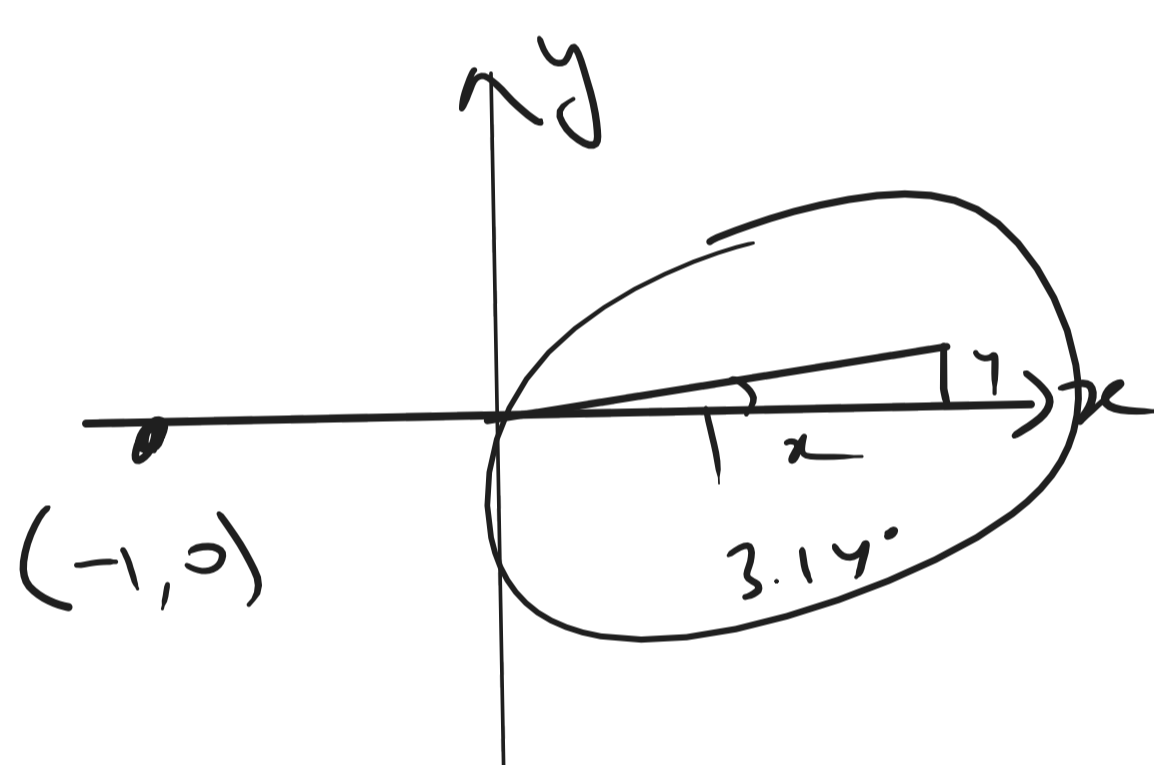


■ $\tan(k\pi + \theta) = \tan \theta$, for all integers k .

$\tan(24\pi)$

$= \tan(\theta \pm k\pi)$

$= \tan(24\pi - 23\pi) = \tan(\pi) = \frac{0}{-1} = 0$.



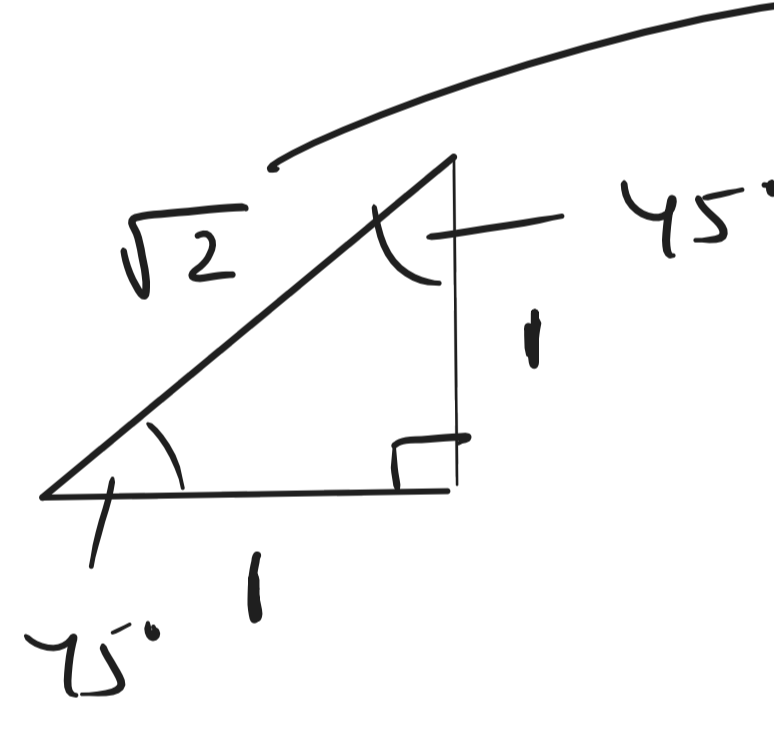
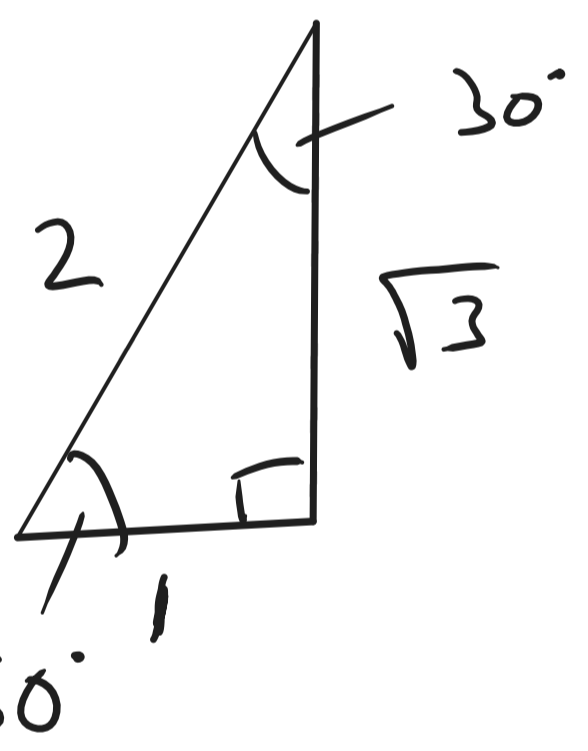
$\tan(3.14) = 0$ radians

$\tan(3.14^\circ) = 0.00027 \approx \frac{1}{250}$

$\sin(0.785) = \frac{\sqrt{2}}{2} \approx 0.7$ radians

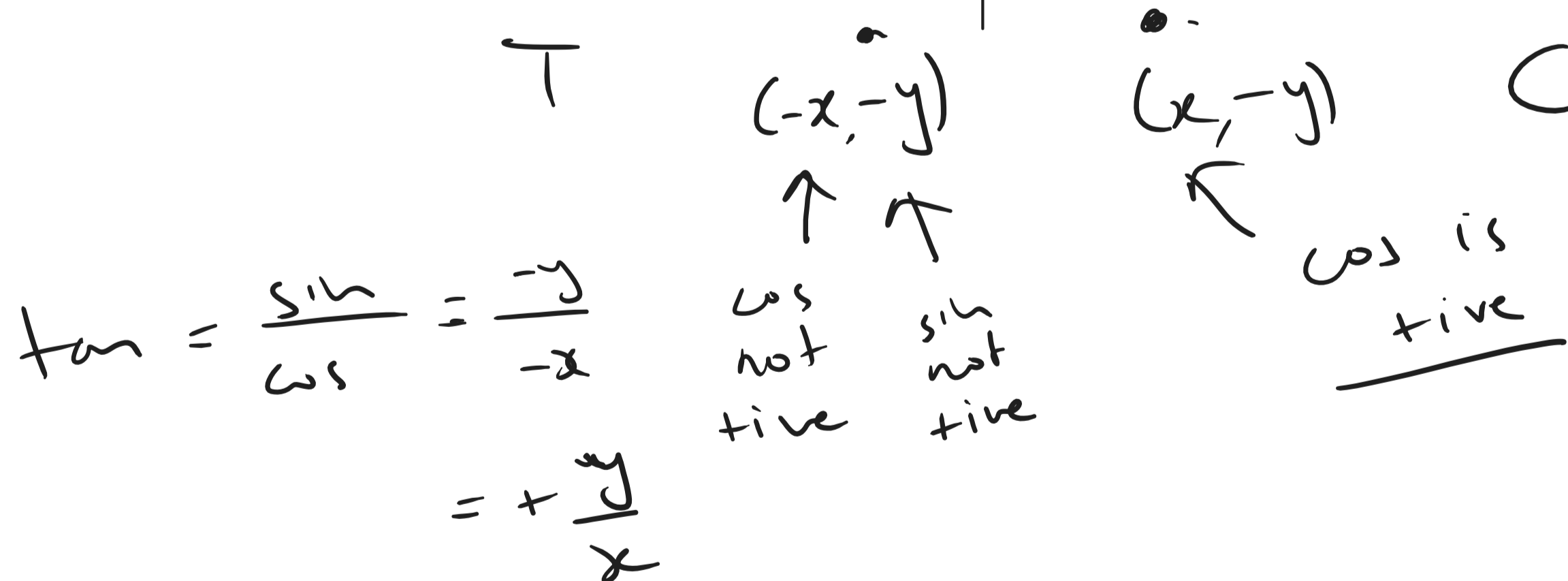
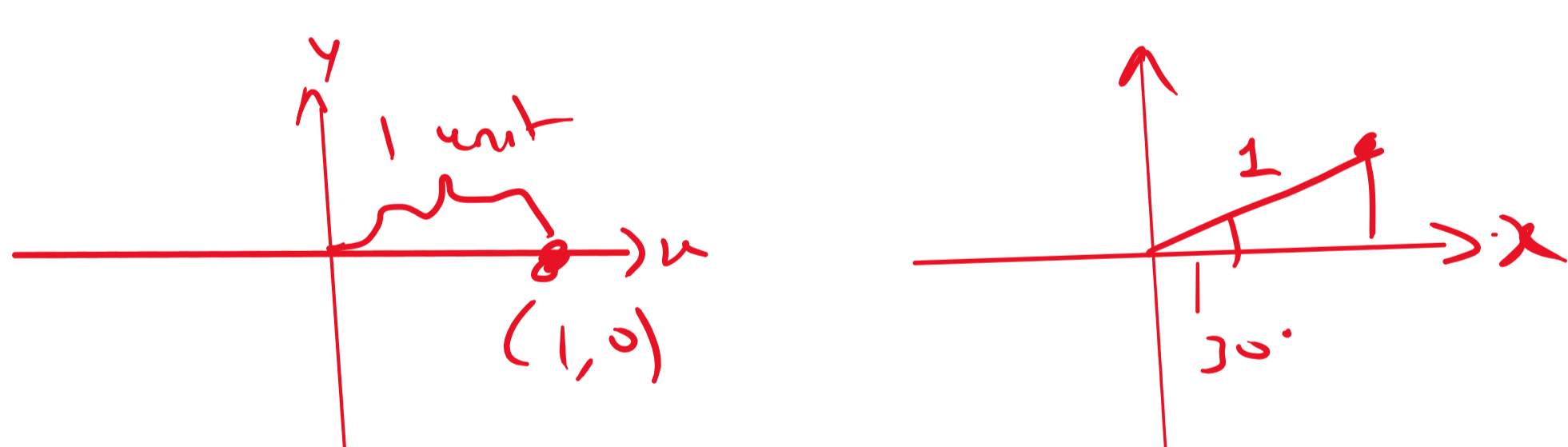
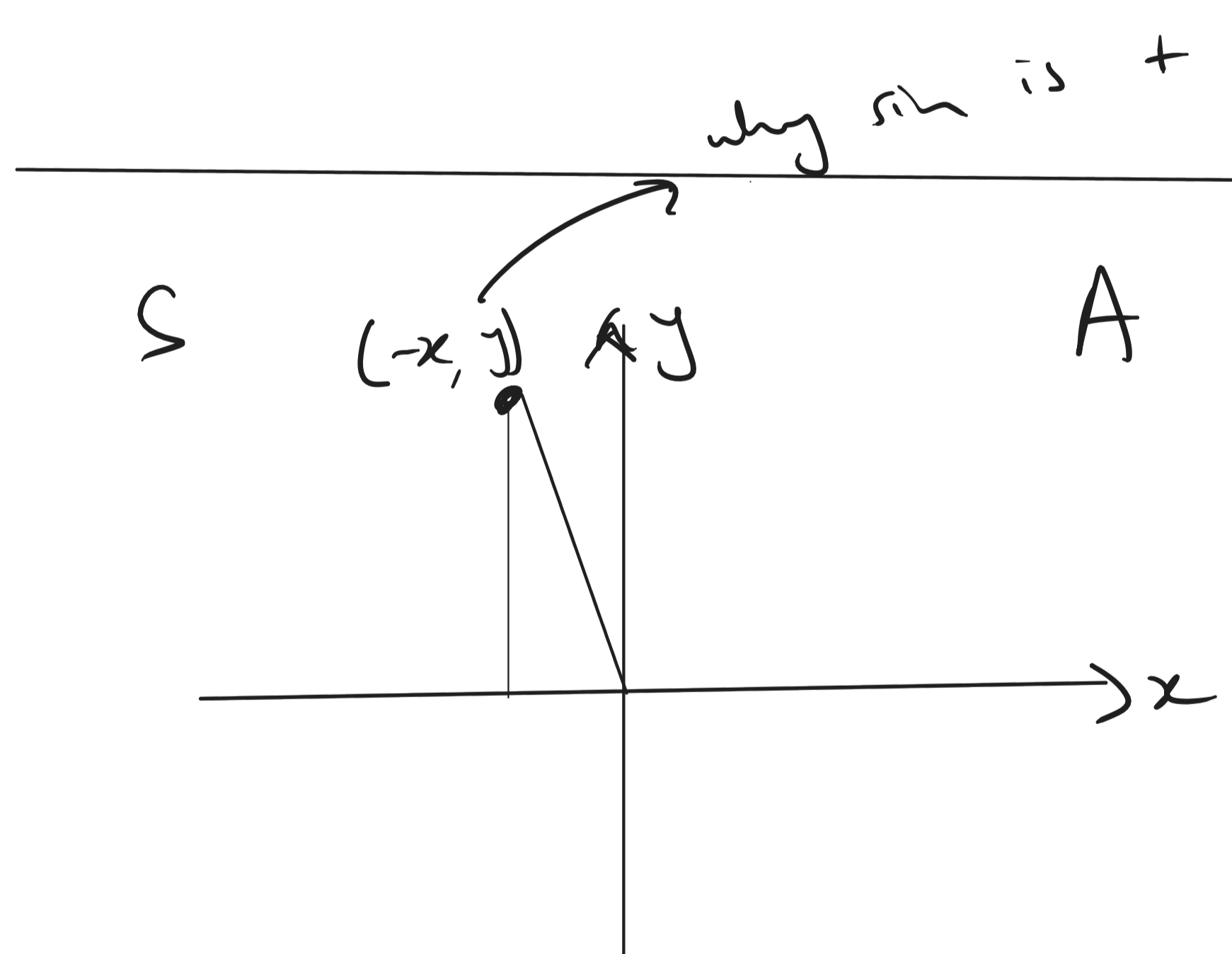
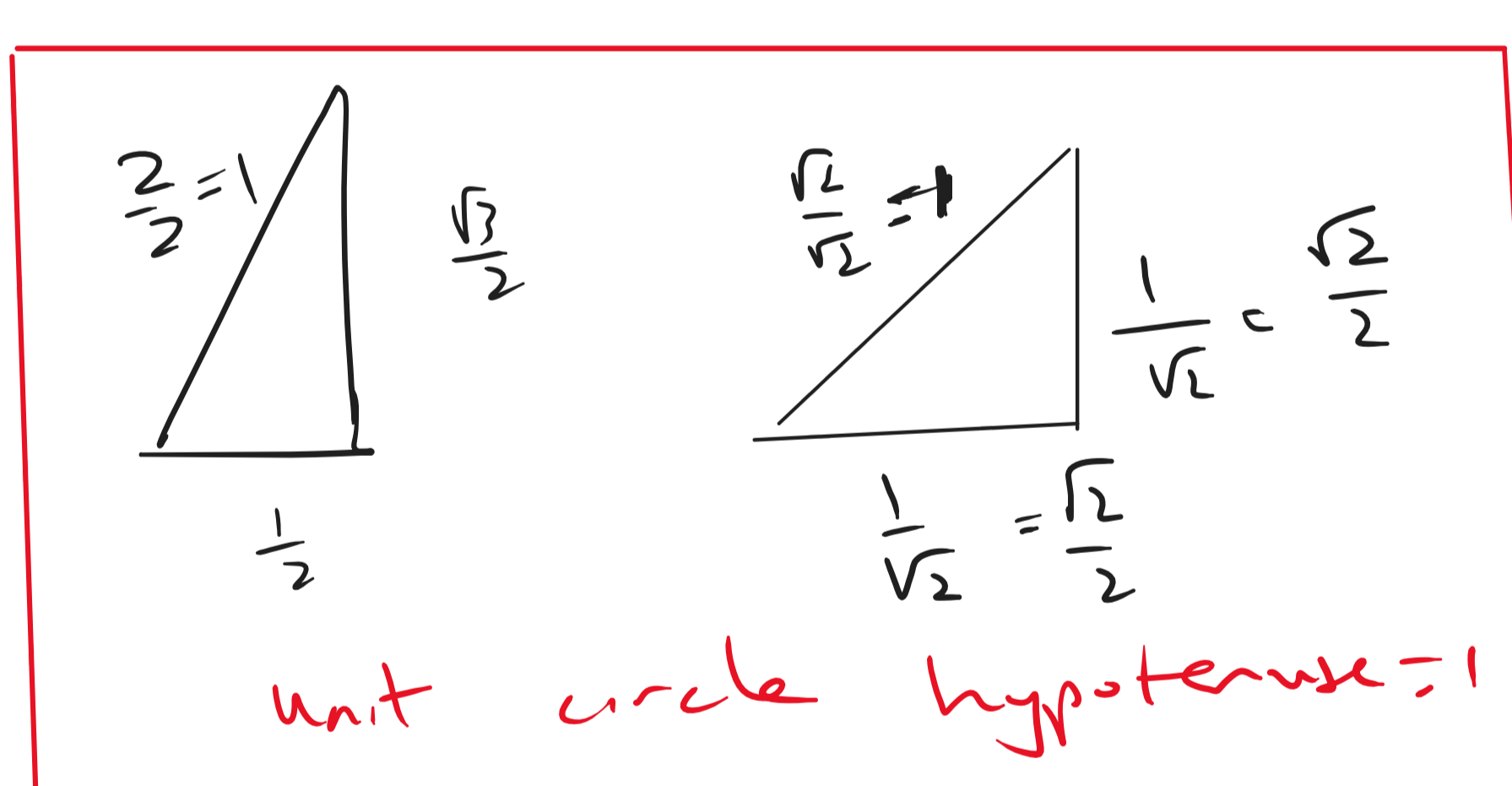
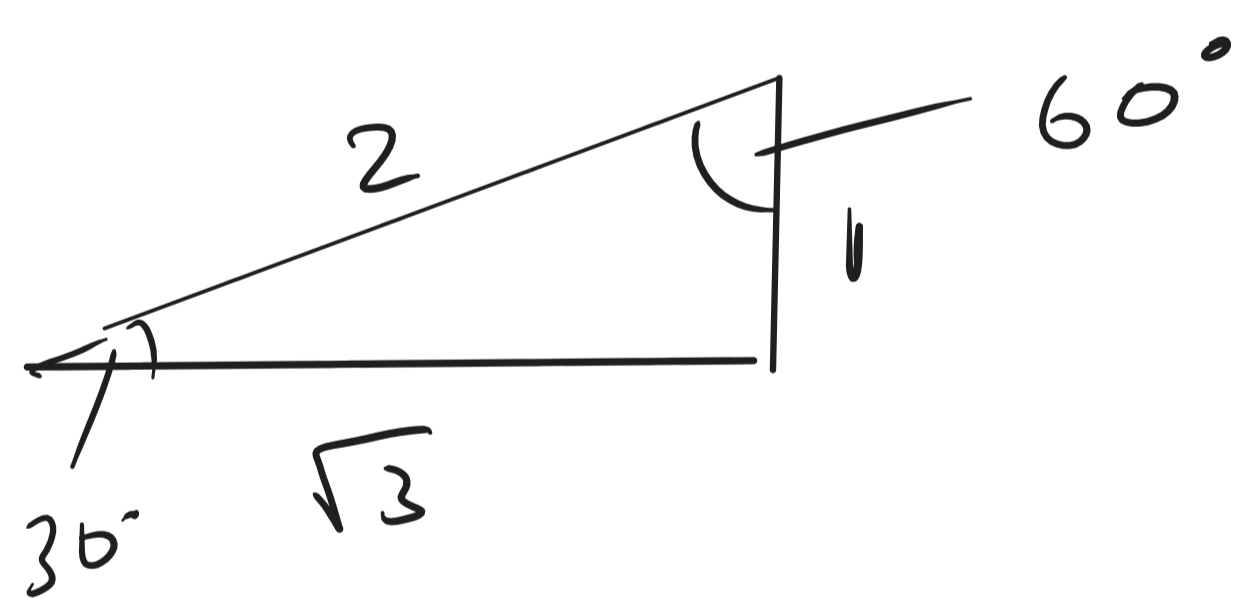
$\sin(0.785) = \text{close to } 0$, degrees

$c^2 = a^2 + b^2$
 $= \sqrt{3^2 + 1^2}$
 $= 3 + 1$
 $= 4$
 $c = \sqrt{4} = 2$



$c^2 = a^2 + b^2$
 $= 1^2 + 1^2$
 $= 1 + 1$
 $= 2$
 $c = \sqrt{2}$

these two triangles must be remembered, needed when finding sin, cos and tan of angles of 30, 45, 60...



Example 5

Evaluate:

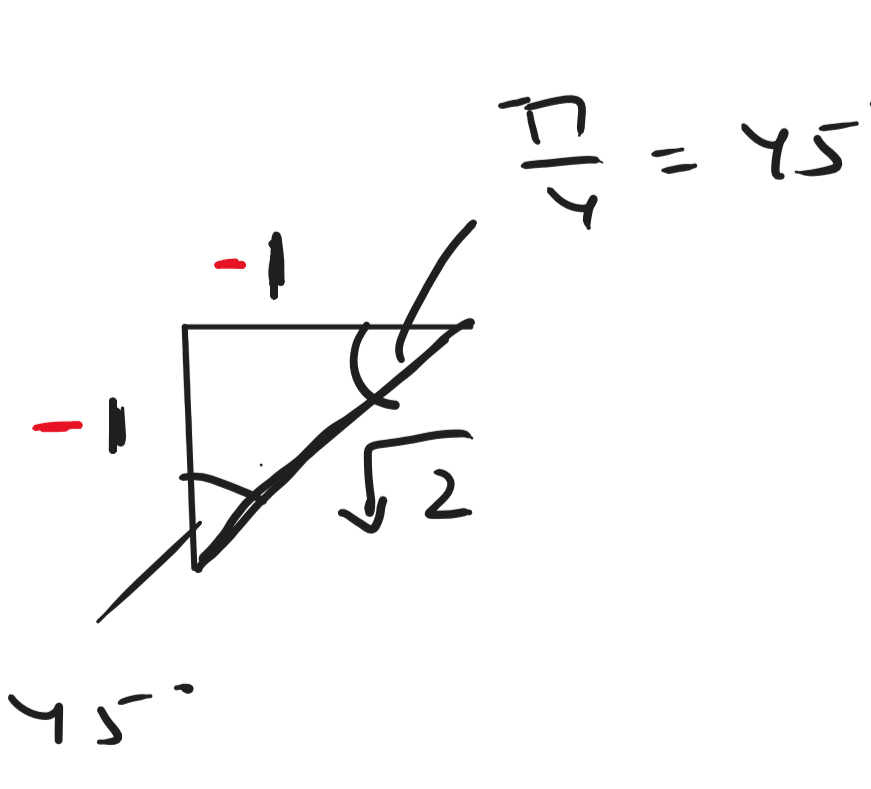
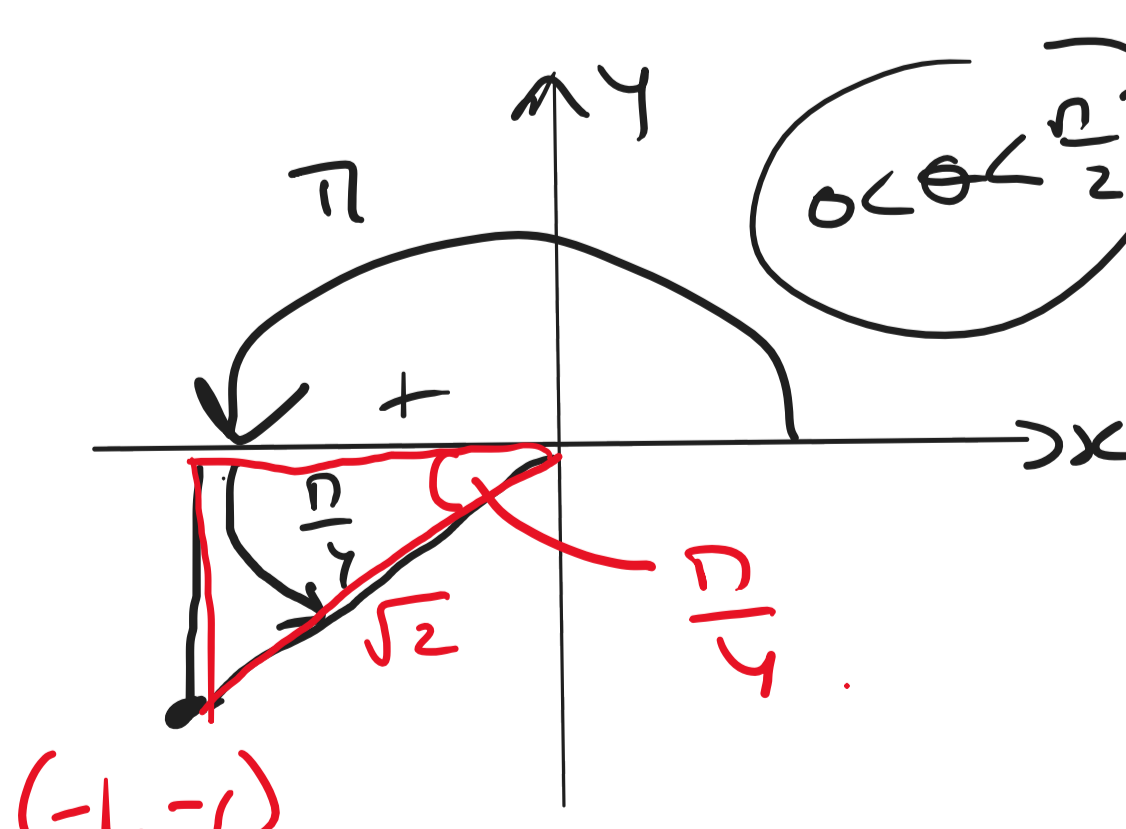
- a** $\cos(\frac{5\pi}{4})$ **b** $\sin(\frac{11\pi}{6})$ **c** $\cos(\frac{200\pi}{3})$ **d** $\tan(\frac{52\pi}{6})$

Solution

- a** $\cos(\frac{5\pi}{4}) = \cos(\pi + \frac{\pi}{4}) = -\cos(\frac{\pi}{4}) = -\frac{1}{\sqrt{2}}$
b $\sin(\frac{11\pi}{6}) = \sin(2\pi - \frac{\pi}{6}) = -\sin(\frac{\pi}{6}) = -\frac{1}{2}$
c $\cos(\frac{200\pi}{3}) = \cos(66\pi + \frac{2\pi}{3}) = \cos(\frac{2\pi}{3}) = \cos(\pi - \frac{\pi}{3}) = -\cos(\frac{\pi}{3}) = -\frac{1}{2}$
d $\tan(\frac{52\pi}{6}) = \tan(8\pi + \frac{2\pi}{3}) = \tan(\frac{2\pi}{3}) = \tan(\pi - \frac{\pi}{3}) = -\tan(\frac{\pi}{3}) = -\sqrt{3}$

$\frac{\pi}{2} = 90^\circ$

$\frac{5\pi}{4}$



$\cos(\frac{\pi}{4}) = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$